2019-1학기 전공튜터링 (5)주차 주요내용 키워드 요약

주제(범위) : 기말고사 대비 라플라스변환 총정리 (예제문제풀이) **작성자 :** 김수진

라플라스변환

의의 : 미분방정식 → 대수방정식

정의 : $f(t) \to F(s)$ $\quad F(s) = \int_{0^-}^{\infty} f(t)e^{-st}dt$

여기서 s는 복소변수

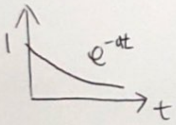
그리고 $f(t)=0$, for $t<0$

선형성 : $\mathcal{L}[af(t)] = a\mathcal{L}[f(t)]$

$\mathcal{L}[f(t)+g(t)] = \mathcal{L}[f(t)] + \mathcal{L}[g(t)]$

의미 : 시간영역 (미분방정식) $\xrightarrow{\mathcal{L}}$ 주파수영역 (대수방정식)

□ 지수함수 : $f(t) = e^{-at}$

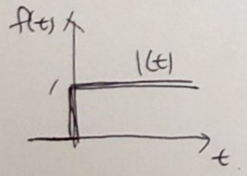


$$F(s) = \int_{0^-}^{\infty} e^{-at}\cdot e^{-st}dt = \int_{0^-}^{\infty} e^{-(s+a)t}dt$$

$$= -\frac{e^{-(s+a)t}}{(s+a)}\Bigg|_0^{\infty} = 0 - \left(-\frac{1}{s+a}\right) = \frac{1}{s+a}$$

$e^{a+b} = e^a \cdot e^b$

□ 단위계단파 함수 : $f(t) = 1(t)$



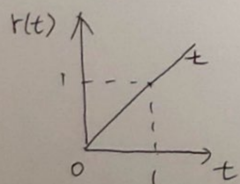
$$F(s) = \int_0^{\infty} 1(t)\cdot e^{-st}dt = \int_0^{\infty} e^{-st}dt$$

$$= -\frac{e^{-st}}{s}\Bigg|_0^{\infty} = 0 - \left(-\frac{1}{s}\right) = \frac{1}{s}$$

$(e^{at})' = ae^{at}$

$\int e^{at}dt = \frac{e^{at}}{a}$

□ 램프함수 : $f(t) = t$



$$F(s) = \int_0^{\infty} te^{-st}dt = \int_0^{\infty}\left(t\frac{e^{-st}}{-s}\right)'dt - \int_0^{\infty}\frac{e^{-st}}{-s}dt$$

$$= t\frac{e^{-st}}{-s}\Bigg|_0^{\infty} - \int_0^{\infty}\frac{e^{-st}}{-s}dt = \frac{1}{s}\int_{0^-}^{\infty} e^{-st}dt = \frac{1}{s^2}$$

- $(f(t)\cdot g(t))' = f(t)'g(t) + f(t)g(t)'$
- $f(t)\cdot g(t)' = (f(t)g(t))' - f(t)'g(t)$

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주제(범위) :

작성자 : 김수진

예제문제1 _ 역변환하여라.

$$F(s) = \frac{s}{(s+\frac{1}{2})^2+1}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s+\frac{1}{2})^2+1}\right\} = \mathcal{L}^{-1}\left\{\frac{s+\frac{1}{2}-\frac{1}{2}}{(s+\frac{1}{2})^2+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+1} - \frac{1}{2}\frac{1}{(s+\frac{1}{2})^2+1}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+1}\right\} - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{(s+\frac{1}{2})^2+1}\right\}$$

$$= e^{-\frac{1}{2}t}\cdot\cos t - \frac{1}{2}e^{-\frac{1}{2}t}\cdot\sin t$$

예제문제2 _ y(t)를 구하여라.

$$y'' + y = 2\cos t, \quad y(0)=3, \quad y'(0)=4$$

$$\mathcal{L}(y''+y) = s^2\cdot\mathcal{L}(y) - sy(0) - y'(0) + \mathcal{L}(y)$$

$$= (s^2+1)\mathcal{L}(y) - 3s - 4$$

$$= \mathcal{L}(2\cos t)$$

$$= 2\cdot\frac{s}{s^2+1}$$

$$(s^2+1)\mathcal{L}(y) = \frac{2s}{s^2+1} + 3s + 4$$

$$\mathcal{L}(y) = \frac{2s}{(s^2+1)^2} + \frac{3s}{s^2+1} + \frac{4}{s^2+1}$$

$$\mathcal{L}^{-1}(Y) = \mathcal{L}^{-1}\left\{\frac{2s}{(s^2+1)^2}\right\} + \mathcal{L}^{-1}\left(\frac{3s}{s^2+1}\right) + \mathcal{L}^{-1}\left(\frac{4}{s^2+1}\right), \quad \left\{\mathcal{L}^{-1}(F'(s))\right\} = -t\cdot f(t)$$

$$y(t) = t\cdot\sin t + 3\cos t + 4\sin t$$